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EFFECT OF ORTHOGONAL INPLANE STRUCTURAL ELEMENTS ON INELASTIC TORSIONAL RESPONSE

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SUMMARY

The elastic torsional stiffness of a structure has important influence on the seismic response of an asymmetric structure, both in the elastic and inelastic range. For elastic structures it is immaterial whether the stiffness is provided solely by structural elements in planes parallel to the direction of earthquake or by a combination of such elements in parallel and orthogonal planes. The issue of how the relative contribution of structural elements in orthogonal planes affects the torsional response of inelastic structures has been the subject of continuing study. Several researchers have noted that structural elements in orthogonal planes reduce the ductility demands in both the flexible and stiff edge elements parallel to the earthquake. Some have noted that the beneficial effect of structural elements in orthogonal planes is more pronounced when such elements remain elastic. These issues are further examined in this paper through analytical studies on the torsional response of single-storey building models. It is shown that, contrary to the findings of some previous studies, the torsional response of inelastic structures is affected primarily by the total torsional stiffness in the elastic range, and not so much by whether such stiffness is contributed solely by structural elements in parallel planes or by such elements in both parallel and orthogonal planes. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: torsional seismic response; single-storey models; torsional stiffness; elastic response; inelastic response; orthogonal inplane structural elements

INTRODUCTION

An asymmetric building structure can be defined as one in which for a purely translational motion the resultants of the resisting forces do not pass through the centres of mass. When subjected to earthquake ground motion, such a structure undergoes both translational and rotational motion even when the earthquake excitation is purely translational. For a structure that remains fully elastic this coupling between translation and torsion may significantly magnify the displacements and forces induced in certain structural elements. For a building structures that is expected to be strained into the inelastic range, torsional motions lead to displacements and ductility demands that may be larger than those in a structure which has similar characteristics,

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but is symmetric and, therefore, experiences no twisting. In the literature the term torsionally unbalanced is often used to refer to structures that are asymmetric. Similarly, a torsionally balanced structure refers to a symmetric structure that is not expected to undergo any twisting under a purely translational excitation.

A number of parameters govern the response of asymmetric buildings, but one that has the most significant effect is the torsional stiffness.[1,2] The torsional stiffness changes continuously as the structure is strained into the inelastic range. In fact, yielding of elements in the structure results in a reduction in both the overall translational and torsional stiffness. However, it is the elastic torsional stiffness that influences both the elastic and the inelastic response. In our discussion, therefore, we will assume that a reference to stiffness, without further qualification, implies a reference to the stiffness in the elastic range. All inplane structural elements (referred to herein as resisting planes, or simply planes), both parallel and perpendicular to the earthquake motion, contribute to the torsional stiffness. It is, therefore, apparent that the resisting planes that are perpendicular to the earthquake motion must be included in any analytical model used to study the earthquake response of asymmetric buildings.

A number of researchers have studied the elastic and inelastic torsional response of single-storey and multi-storey building models. However, certain questions, particularly those related to the inelastic torsional behaviour, have not been adequately addressed. One such issue is the effect of orthogonal planes on the inelastic torsional behaviour. Recent studies on torsional response and the effect of orthogonal planes include those by Humar and Kumar,[1,2] Goel and Chopra,[3] Correnza *et al.*,[4,5] Chandler *et al.*,[6] and Paulay.[7,8] Many of the conclusions arrived at in these studies are at variance with each other, adding to the confusions surrounding the issue.

Goel and Chopra[3] compared the inelastic torsional response of two single-storey asymmetric rectangular building models with identical torsional stiffnesses but with and without orthogonal planes. In a direction perpendicular to the direction of earthquake both models had non-uniform distribution of stiffness, so that the centre of stiffness was offset from the geometric centre, which was also the centre of mass. The edge of the building on the same side of the centre of mass as the centre of stiffness was referred to as the stiff edge, the other edge being called the flexible edge. The researchers concluded that: (1) peak responses of short period, acceleration sensitive systems were influenced significantly by the contribution to torsional stiffness from orthogonal planes; (2) in a system with orthogonal planes, the parallel plane located at the flexible edge experienced smaller deformations, while the plane on the stiff edge underwent larger deformations in comparison to corresponding planes in a similar system without orthogonal planes; and (3) ductility demand for a system with orthogonal planes was smaller at the flexible edge and larger at the stiff edge than that in a similar system without orthogonal planes. Goel and Chopra's results were based on models that had one specific value of eccentricity between the centre of mass and the centre of stiffness (20 per cent of the plan dimension perpendicular to the direction of the earthquake), and equal uncoupled torsional and translational frequencies. Further, the results they obtained were from analytical studies in which an idealized ground motion was applied in only one direction and the orthogonal planes were assumed to remain elastic.

Correnza *et al.*[4,5] studied the response of single-storey models with and without orthogonal planes. They also compared the response of models with orthogonal planes subjected to unidirectional earthquakes to the response of the same models when subjected to bidirectional earthquakes. They concluded that the orthogonal planes affected significantly the response of torsionally unbalanced systems, particularly in the short- and medium-period range. Their studies

also showed that in carrying out a non-linear dynamic analysis of models with orthogonal planes, it was important to include both horizontal components of the ground motion.

Paulay's studies,[7,8] related to the ductility demand in an asymmetric building, were based on a simple static plastic mechanism analysis of single-storey models. He showed that in a torsionally unrestrained system, that is, one in which orthogonal elastic elements were not available to resist torsion, the ductility demand, particularly in the edge elements, could be excessive. The elements most likely to be critical were those that had the smallest yield displacement.

On the basis of analytical studies of elastic and inelastic response Humar and Kumar[1,2] concluded that the single most important parameter governing the torsional response was the ratio of the uncoupled elastic torsional frequency to the uncoupled elastic translational frequency, or equivalently, the ratio of torsional to translational stiffness in the elastic range. It was immaterial whether the torsional stiffness was provided only by the planes parallel to the direction of the earthquake, or by both the parallel and the orthogonal planes. The present study explores this further by addressing the following issues: (1) effect of the variation in elastic torsional stiffness of orthogonal planes while the overall elastic torsional stiffness of the system is held constant, (2) effect of yielding in the orthogonal planes on the torsional behaviour of the system, and (3) the effect of the uncoupled elastic translational period of the building model studied.

Some of the results presented in this study as well as those presented by Humar and Kumar[2] are at variance with the results obtained by Correnza *et al.*[4,5] and Paulay.[7,8] The reasons for the variations are explored in the present study.

A majority of the results presented here are obtained from the linear and non-linear dynamic analyses of a single storey, mono-symmetric building model for its response to a set of 12 earthquake motions.

BUILDING MODELS STUDIED

The single-storey building model used in this study is shown in Figure 1(a). The model is rectangular in plan and has a rigid floor deck. The floor mass m is assumed to be uniformly distributed, so that the centre of mass (CM) lies at the geometric centre of the floor deck. The model has three resisting planes parallel to the y-axis and two planes parallel to the x-axis. The resisting planes may be comprised of cantilever walls, braced frames, moment resisting frames, or a combination. The distribution of stiffness among the y-direction planes is such that the centre of stiffness is eccentric with respect to the geometric centre, which is also the centre of mass. The distance between the centre of stiffness and the centre of mass is called the static eccentricity and is denoted by e. The centre of stiffness is also referred to as the centre of rigidity, and is, therefore, denoted here by CR. It may be noted that in the elastic range the resisting force in any individual plane is proportional to its stiffness. Consequently, for a purely translational motion the resultant of all resisting forces passes through the centre of stiffness; the centre of resistance thus coincides with the centre of stiffness. The centre of resistance, however, moves away from its position in the elastic range as one or more planes yield.

The resisting planes parallel to the x-axis are identical and are symmetrically placed. The dimension of the building along the x-axis is b and along the y-axis is a. The plan aspect ratio is defined as $\alpha = a/b$. The mass radius of gyration about the CM is $r = \sqrt{(a^2 + b^2)/12}$. The

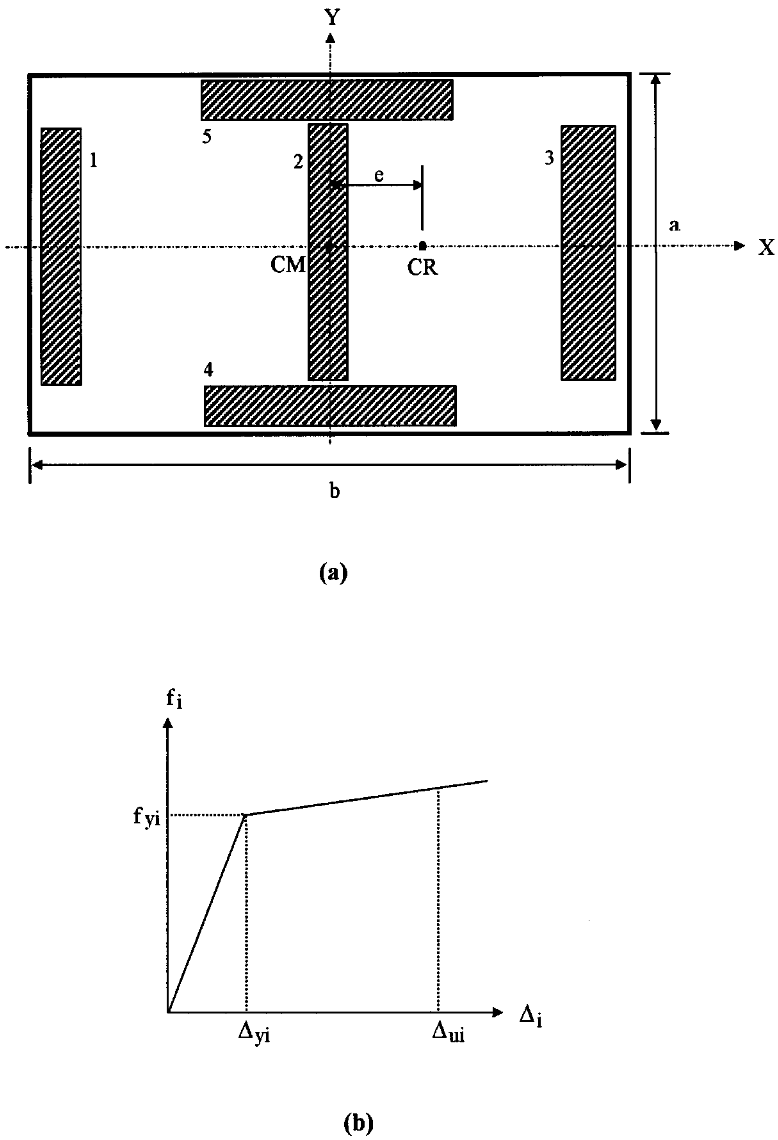


Figure 1. (a) Plan view of a five-plane single-storey monosymmetric building model, (b) force–displacement relationship for plane i

uncoupled elastic translational frequency ω_y and the uncoupled elastic torsional frequency ω_θ are defined as

$$\omega_y = \sqrt{K_y/m} \tag{1a}$$

$$\omega_\theta = \sqrt{K_{\theta R}/mr^2} \tag{1b}$$

where K_y is the sum of the elastic stiffness of planes in the y-direction and $K_{\theta R}$ is the torsional stiffness about the centre of stiffness (CR). The uncoupled frequency ratio is defined as

$$\Omega_R = \omega_\theta/\omega_y = \sqrt{\frac{K_{\theta R}}{r^2 K_y}} \tag{2}$$

For the purpose of obtaining numerical results the following data are used for all the models studied: $b = 36$ m, $m = 400$ t, $mr^2 = 54\,000$ t m^2. A wide range of building models with different combination of e/b and Ω_R values is selected for study.

To study the effect of orthogonal planes, a parameter γ is introduced. It is defined as the ratio of the torsional stiffness (about CR) of planes parallel to y-axis to the overall torsional stiffness of the system $K_{\theta R}$. A lower value of γ indicates a higher contribution from orthogonal elements towards the total torsional stiffness of the model.

EARTHQUAKE MOTIONS

A set of 12 earthquake records, each having ground acceleration data in two orthogonal horizontal directions, is selected. The characteristics of the selected earthquakes are presented in Table I. Of the two components of an earthquake, the component that has the higher peak acceleration is referred to as the major direction component; the other component being referred to as the minor direction component. The peak values of any ground parameter (acceleration, velocity, or displacement) in the two components rarely occur simultaneously, the temporal distance between them varying from record to record. The average value of the ratio of peak ground acceleration in the minor direction component of an earthquake to that in the major direction component is found to be 0·88. In the dynamic time history analysis carried out in this study, the major direction component of each earthquake is scaled to a peak ground acceleration of $0{\cdot}3g$, while the minor direction component is scaled to a peak ground acceleration of $0{\cdot}88 * 0{\cdot}3g = 0{\cdot}264g$.

DETAILS OF ANALYSIS

In the Torsionally Unbalanced (TUB) model of the building shown in Figure 1(a), the mass centre (CM) coincides with the geometric centre of the building, while the centre of stiffness (CR) is offset by a distance e. The stiffnesses of the individual planes in the model are determined once the values of the eccentricity e/b, the frequency ratio Ω_R and the translational period of the building have been selected. For the presentation of analytical results, a reference Torsionally Balanced (TB) model is defined as the one in which the CM is shifted away from its position at the centre, so as to coincide with CR, but the values of T_y and Ω_R are identical to those in the TUB model.

The force–displacement relationship for each resisting plane in both the TB and TUB models is assumed to be bilinear, with a post-yield stiffness equal to 5 per cent of the initial elastic stiffness, as shown in Figure 1(b). The yield strength of plane i is denoted by f_{yi}, the yield displacement by Δ_{yi}, and the maximum displacement attained during a response history by Δ_{ui}. The ductility demand for plane i is given by $\mu_i = \Delta_{ui}/\Delta_{yi}$. In the TB model, the system design strength corresponding to elastic response in the y-direction, V_{ey}, is obtained from a design response

Table I. Description and peak ground motion parameters for earthquake records

No.	Earthquake	Date	Magn.	Site	Epic. dist. (km)	Comp.	Max. acc. (cm/s^2)	Max. vel. (cm/s)	a/v
1	Nahanni Aftershock Canada	12/23/85	6·9	Iverson Northwest Territories	7	280	1319·1	45·06	2·98
2	Nahanni Aftershock Canada	12/23/85	6·9	Battlement Creek, N.W. Territories	21	360	190·2	3·43	5·65
3	Nahanni Aftershock Canada	12/25/85	5·7	Battlement Creek, N.W. Territories	18	360	103·4	1·05	10·04
4	Miyagi Prefecture Japan	6/12/78	6·3	Ofunato Harbor Jetty	103	E41S	222·1	14·10	1·61
5	Michoacan Mexico City	9/19/85	8·1	LA Union	80	N00E	162·8	20·34	0·82
6	Michoacan Mexico City	9/19/85	8·1	La Villita	40	N00E	121·0	16·11	0·77
7	ADAK U.S.A.	5/2/71	6·8	ADAK Naval Base (Hand dig)	69	N90E	183·7	6·35	2·95
8	Sitka U.S.A.	7/30/72	7·6	Sitka Magnetic Observatory	48	West	91·3	9·32	1·0
9	San Fernando California	2/9/71	6·5	Lake Huges Array 4	28	S69E	168·2	5·75	2·98
10	Coyote Lake U.S.A.	8/6/79	5·8	Gilory Array 6 San Ysidro	10	230	409·0	43·80	0·95
11	Loma Prieta California	10/18/89	7·1	Appel Array 9 Crystal Springs Reservoir	62	137	115·1	17·13	0·68
12	Loma Prieta California	10/18/89	7·1	Calaveras Array Cherry Flat Reservoir (left abutment)	42	360	78·2	8·73	0·91

spectrum corresponding to the translational period of vibration T_y. The total yield strength of the TB model in the y-direction is now taken as $V_{y0} = V_{ey}/R$, where the force modification factor R is equal to 4. This value of R is based on a system ductility capacity $\mu = 4$ and the assumption that the maximum total displacements imposed by the design earthquake in the elastic and the inelastic systems are the same. The yield strength f_{yi} of an individual plane in the TB model is taken as being proportional to its stiffness k_i, so that

$$f_{yi} = \frac{k_i}{K_y} V_{y0} \tag{3}$$

When the yield strengths are defined as in equation (3), the yield displacement Δ_{yi} is the same for all the planes. The TB model undergoes pure translational motion for the entire duration of its response to a y-direction earthquake. All y-direction planes yield simultaneously and have identical maximum displacement. Thus, the ductility demand on each plane is the same as the system ductility. Individual planes should, therefore, be detailed to have a ductility capacity equal to or greater than the system ductility used in deriving the design forces.

An equivalent static method based on the concept of design eccentricity is used to obtain the yield strengths of planes in the y-direction of TUB model. The following design eccentricities suggested recently by Humar and Kumar[1,2] are used

$$e_{d1} = e + 0{\cdot}1b \tag{4}$$

$$e_{d2} = e - 0{\cdot}1b, \qquad \Omega_{\mathrm{R}} \geqslant 1{\cdot}0 \tag{5a}$$

$$\mathrm{e}_{d2} = -0{\cdot}1b, \qquad \Omega_{\mathrm{R}} < 1{\cdot}0 \tag{5b}$$

where e is the eccentricity between the CM and CR.

Using Equations (4) and (5), the following expressions are obtained for the yield strength of planes in the y-direction:

$$f_{y1} = V_{y0}\frac{k_1}{K_y}\left[1 + \frac{1}{\Omega_{\mathrm{R}}^2}\left(\frac{b}{r}\right)^2\frac{e_{d1}}{b}\left(\frac{e}{b} + 0{\cdot}5\right)\right] \tag{6}$$

$$f_{y2} = V_{y0}\frac{k_2}{K_y}\left[1 + \frac{1}{\Omega_{\mathrm{R}}^2}\left(\frac{b}{r}\right)^2\frac{e_{d1}}{b}\left(\frac{e}{b}\right)\right] \tag{7}$$

$$f_{y3} = V_{y0}\frac{k_3}{K_y}\left[1 - \frac{1}{\Omega_{\mathrm{R}}^2}\left(\frac{b}{r}\right)^2\frac{e_{d2}}{b}\left(0{\cdot}5 - \frac{e}{b}\right)\right] \tag{8}$$

$$f_{y3} = V_{y0}\frac{k_3}{K_y}\left[1 - \frac{1}{\Omega_{\mathrm{R}}^2}\left(\frac{b}{r}\right)^2\frac{e_{d1}}{b}\left(0{\cdot}5 - \frac{e}{b}\right)\right] \tag{9}$$

In determining f_{y3}, the larger of the absolute values obtained from Equations (8) and (9) is used.

The properties of the x-direction planes in the TB and TUB models are identical. The total stiffness of these planes is determined from the selected x-translation period. This stiffness is equally divided among the two planes. The total strength corresponding to elastic response of elements in the x direction, V_{ex}, is obtained from the design response spectrum corresponding to the x-direction translation period. The total yield strength in the x-direction is taken as $V_{ex}/4$ and each plane is assigned half of this strength.

To obtain the design elastic spectrum, each of the 12 major component records is scaled so that its peak acceleration is $0{\cdot}3g$. The average of the elastic spectra for the scaled records and for 5 per cent damping is taken as the design response spectrum. The same design spectrum is used for both x- and y-directions as is the standard practice in seismic design.

Dynamic time history analyses are carried out on both the TB and TUB models for the earthquake records scaled as described earlier. It should be noted that the design eccentricities given by Equations (4) and (5) include a provision for possible accidental torsion. In order to verify whether these provisions lead to adequate design, the dynamic analyses carried out on the building models must also include the effect of accidental eccentricity. Recently, De La Llera and

Chopra[9] have suggested that the effect of accidental eccentricity can be taken into account reasonably by shifting the mass centres by $\pm 0{\cdot}05b$ in the models being analysed. The TUB models are therefore modified by moving the centre of mass $\pm 0{\cdot}05b$ along the axis, and the higher of the two responses obtained from corresponding modified models is considered for each element.

In the analysis of each pair of TUB models, it is assumed that earthquake excitation is applied simultaneously along both x- and y-directions. To demonstrate the effect of yielding in orthogonal elements on torsional response of the system, the above-mentioned set of analyses is repeated with the difference that the strength of orthogonal planes is taken to be very high. This is done to ensure that the orthogonal planes remain elastic.

The maximum ductility demand in a resisting plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b. As stated earlier, for the TB model the ductility demand is identical for all planes and is equal to the system ductility demand. This is not true for the TUB model. The ratio of ductilities $r_\mu = \mu_u/\mu_b$ provides a measure of the effect of torsional motion on individual planes. A mean value of the ductility ratios, obtained for the set of 12 earthquakes, is denoted by $\bar{r}_\mu$. If this ratio is less than 1 for all planes, the design is as safe as that for the TB model.

RESULTS OF RESPONSE ANALYSES

Results are presented first for building models with the uncoupled period of translation in y-direction equal to 1·0 s., and that in the x-direction equal to 0·5 s.

The mean ductility ratio for the flexible edge element $\bar{r}_{\mu f}$ is plotted as a function of e/b in Figures 2(a)–2(g), for different values of Ω_R and γ. It should be noted that $\gamma = 1$ represents a building model without orthogonal elements. Also plotted in these graphs are the responses of building models for which orthogonal planes are designed to remain elastic. The value of $\bar{r}_{\mu f}$ is less than 1 in all cases implying that the ductility demand on the flexible edge plane in a torsionally unbalanced building, designed according to the procedure suggested here, is substantially less than that in the associated balanced building.

The mean value of ductility ratio for the stiff edge element $\bar{r}_{\mu s}$ is plotted against e/b in Figures 3(a)–3(g). Here again, $\gamma = 1$ represents a building model without orthogonal elements and a lower value of γ indicates a higher contribution of orthogonal elements towards total torsional stiffness of the system. Ductility ratio $\bar{r}_{\mu s}$ is also found to be less than 1 for most of the models studied.

Effect of orthogonal planes

Figures 2(a)–2(g) indicate that the presence of orthogonal planes reduces the ductility demand at the flexible edge of the building. In general, for a smaller value of γ, i.e. for a higher contribution of orthogonal planes, this effect is higher. However, the reduction in ductility demands on account of the presence of orthogonal planes is quite modest (<10 per cent) and may be considered insignificant for practical purposes. This observation holds for all values of γ. As would be expected, the reduction is more significant when the orthogonal planes remain elastic. This is discussed in greater detail later in this paper.

Figures 3(a)–3(g) indicate that the presence of orthogonal planes increases the ductility demand at the stiff edge of the building for $\Omega_R = 1$ but reduces this ductility demand for $\Omega_R = 1{\cdot}25$ and $1{\cdot}50$. For a lower value of γ, this effect is more pronounced. Here again, the difference in ductility demands for models with different γ values is quite small (< 10 per cent) and may be considered insignificant.

It is interesting to note that when the orthogonal planes are designed to remain elastic, the ductility demand at stiff edge planes increases further, and in most cases is higher than that in systems in which there are no orthogonal planes. This is dealt with greater detail in the following section.

The results presented so far tend to indicate that for a single-storey building system in which orthogonal planes as well as parallel planes yield during an earthquake, the ductility demand of an edge plane depends more or less on the total torsional stiffness of the building and not on what part of it is contributed by orthogonal planes. The effect of γ on the inelastic torsional response of single storey building models is thus quite small.

Effect of yielding in orthogonal planes

Figures 2(a)–2(g) indicate that the ductility demand at the flexible edge of a building model reduces if the orthogonal planes remain elastic during an earthquake. This is observed to be true for all values of Ω_R and γ. This reduction is quite small (< 5 per cent) for $\Omega_R = 0{\cdot}75$ and $1{\cdot}0$, but is a little higher (up to 15 per cent) for $\Omega_R = 1{\cdot}25$ and $1{\cdot}50$, specially for large eccentricity values.

Figures 3(a)–3(g) indicate that, for $\Omega_R = 0{\cdot}75$ and $1{\cdot}0$, the ductility demand at the stiff edge of a building model in which orthogonal planes remain elastic during an earthquake, is more or less the same as that for models with yielding orthogonal planes. For $\Omega_R = 1{\cdot}25$ and $1{\cdot}50$, non-yielding orthogonal planes lead to a decreasing ductility demand (up to 5 per cent) for low values of e/b ($< 0{\cdot}1$), but an increased ductility demand (up to 20 per cent) for higher values of e/b. This is observed to be true for all values of γ.

The decrease in ductility demands at the flexible edge planes and increase in the ductility demand at the stiff edge planes of a building system, can be explained as follows. The total response of a lateral load-resisting plane in a single-storey building model arises from a combination of rotational and lateral responses of the system. The rotational motion adds to the lateral displacement at the flexible edge but compensates the lateral displacement at the stiff edge. Except for systems that are torsionally very flexible ($\Omega_R < 0{\cdot}75$), the total displacement at the stiff edge reduces as a result of torsion. If the orthogonal planes remain elastic, they provide a relatively higher torsional resistance in comparison to the case when these orthogonal planes are yielding. As a consequence, the presence of elastic orthogonal planes reduces the torsional response of the system. The result is a reduction in the total response at the flexible edge and an increase in the total response at the stiff edge.

Analyses similar to those described in the previous paragraphs are repeated for models with y-direction period equal to $0{\cdot}5$ s. and the x-direction period equal to $0{\cdot}2$ s. The results, not presented here, indicate a pattern very similar to the one seen in Figures 2 and 3. All the conclusions drawn from the results of previous analyses hold true for shorter-period building models analysed.

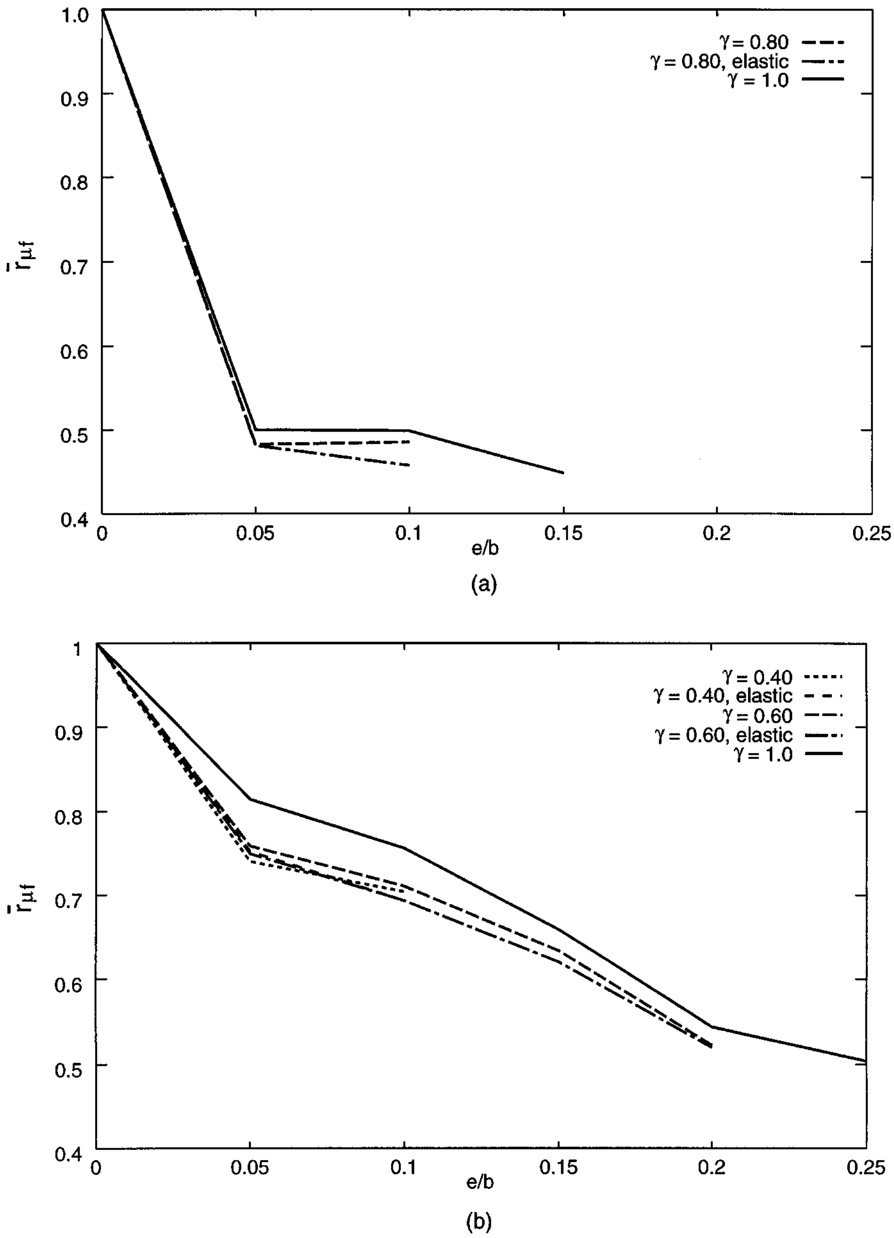
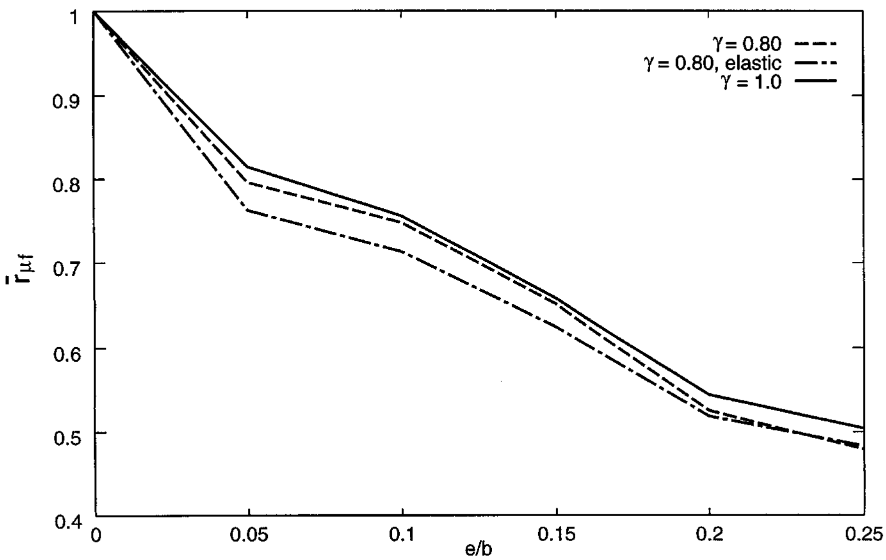
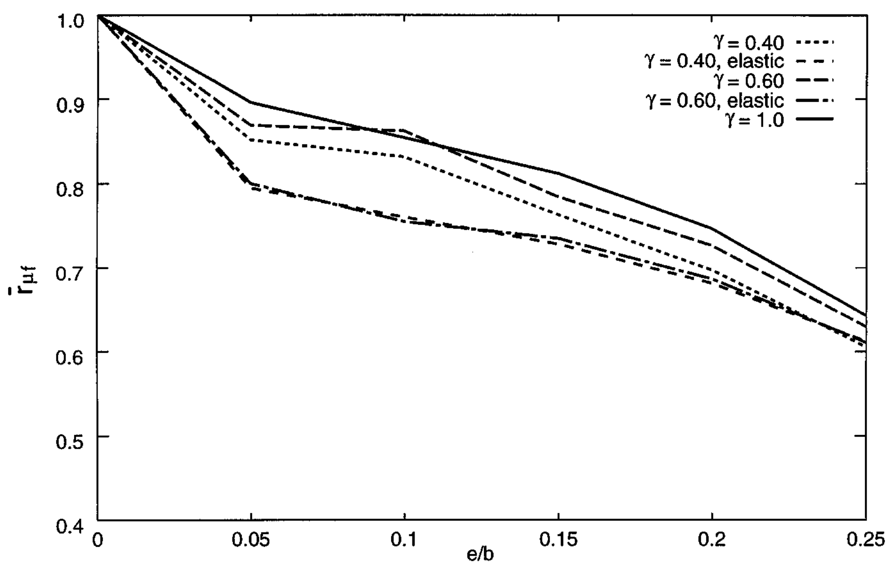


Figure 2. Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1{\cdot}0$ s. (a) $\Omega_R = 0{\cdot}75$, $\gamma = 0{\cdot}8$, 1·0; (b) $\Omega_R = 1{\cdot}00$, $\gamma = 0{\cdot}4$, 0·6, 1·0; (c) $\Omega_R = 1{\cdot}00$, $\gamma = 0{\cdot}8$, 1·0; (d) $\Omega_R = 1{\cdot}25$, $\gamma = 0{\cdot}4$, 0·6, 1·0; (e) $\Omega_R = 1{\cdot}25$, $\gamma = 0{\cdot}8$, 1·0; (f) $\Omega_R = 1{\cdot}50$, $\gamma = 0{\cdot}4$, 0·6, 1·0; (g) $\Omega_R = 1{\cdot}50$, $\gamma = 0{\cdot}8$, 1·0

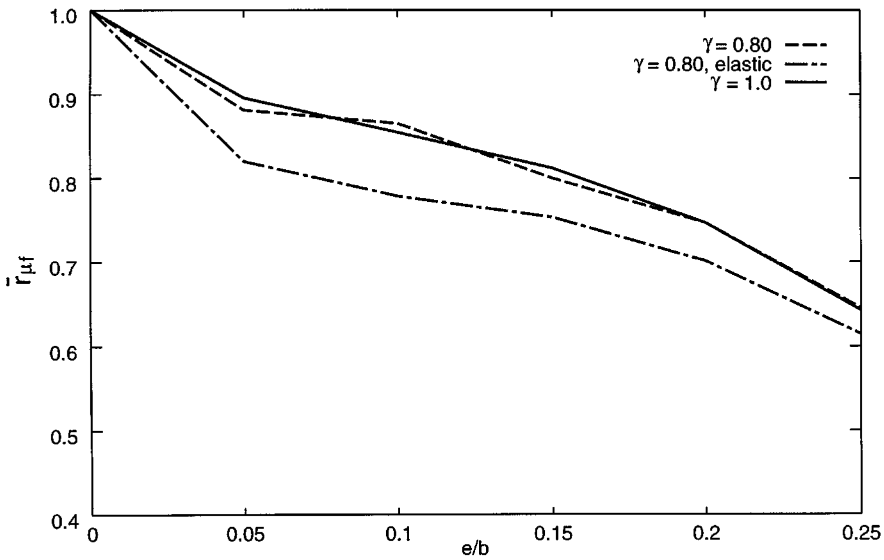


(c)



(d)

Figure 2. Continued



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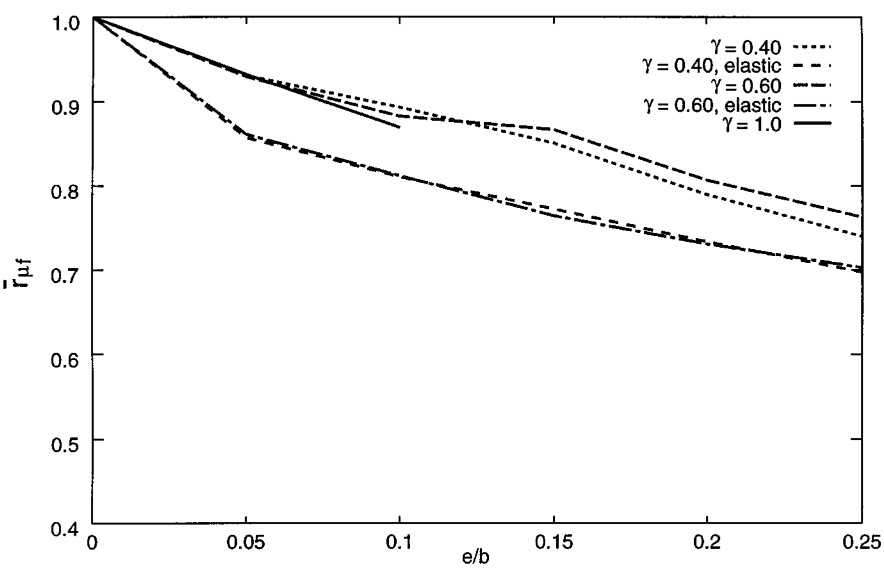

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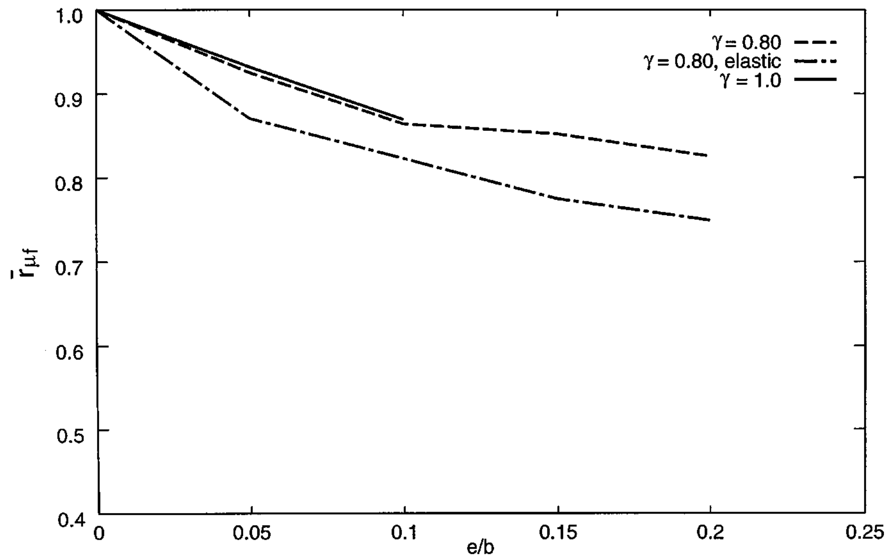


(g)

Figure 2. Continued

PERIOD DEPENDENCE

Results presented in the preceding section indicate that the effect of orthogonal planes on the seismic response of asymmetric buildings is similar for short period of 0·5 s. and medium period of 1·0 s. This is likely to be true of all periods, so that the effect of orthogonal planes can be considered period independent. This is examined further in the present section.

The building model studied is the same as shown in Figure 1(a).. The eccentricity is chosen as 0.3b. For the model in which there are no orthogonal planes, referred to as model A, it is assumed that $k_1 = k_2$. This assumption yields $k_1 = k_2 = 0{\cdot}1333K_y$, and $k_3 = 0{\cdot}7333K_y$. For the model in which there are two orthogonal planes, referred to as model B, it is assumed that $K_x = K_y$. Further, the torsional stiffness of models A and B are assumed to be equal. This is achieved by reducing the stiffness of planes 1 and 3 by equal amounts and increasing the stiffness of plane 2 appropriately so that K_y remains unchanged. Model B has the following properties, $k_1 = 0{\cdot}00833K_y$, $k_2 = 0{\cdot}38333K_y$, $k_3 = 0{\cdot}60833K_y$ and $k_4 = k_5 = 0{\cdot}5K_y$. The total stiffness K_y is selected to provide a specified period. The period is varied over a range of values.

The design earthquake is taken as the North–South component of El-Centro 1940 earthquake. For the present study, a smoothened Newmark–Hall spectrum corresponding to 5 per cent damping and median level is used for obtaining the design strength. A value of $R = 5$ is chosen so that the design strength is 1/5th of elastic base shear obtained from the smoothened spectrum. The strengths of individual planes in the TB and TUB models are now distributed by the procedure described earlier. Thus, Equation (3) is used to determine the yield strength for planes in the TB model, while Equations (6)–(9) are used to obtain the yield strengths of planes in the TUB model. The modeling details and the design spectra described in the previous paragraphs

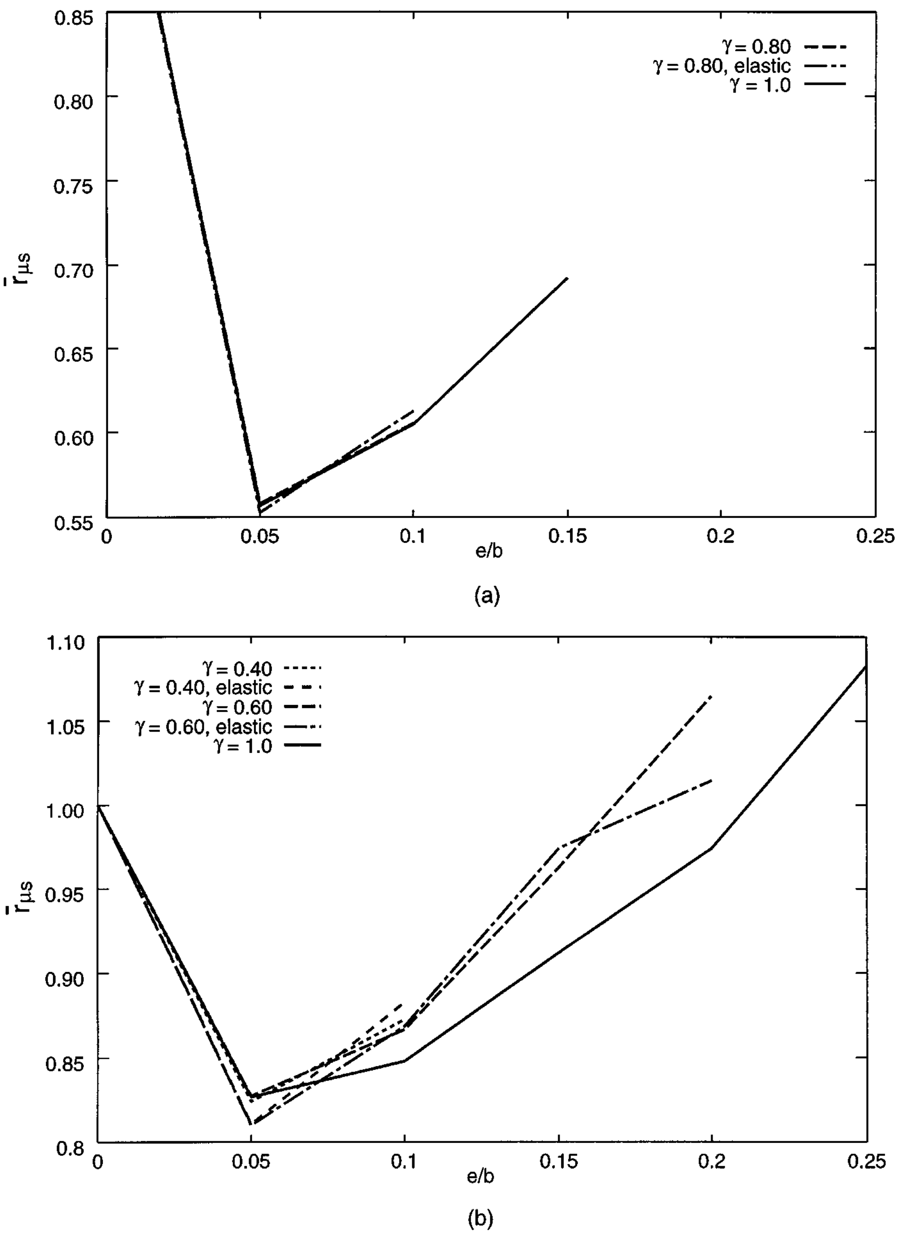
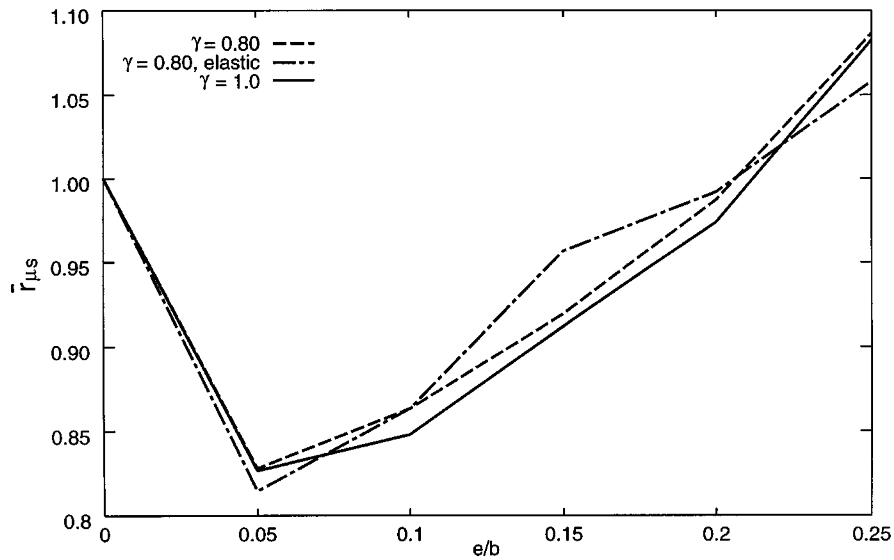
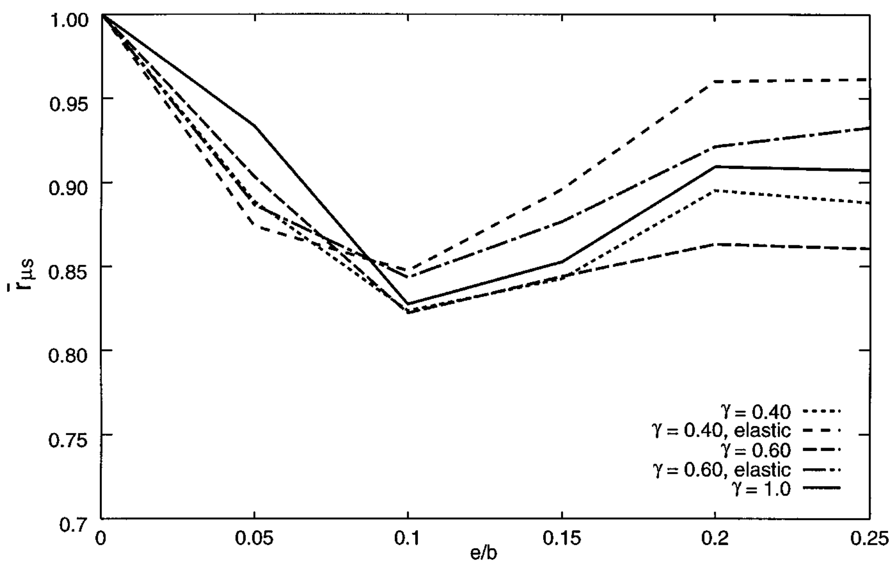


Figure 3. Ratio of stiff edge ductility demand in a five plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1{\cdot}0$ s. (a) $\Omega_R = 0{\cdot}75$, $\gamma = 0{\cdot}8$, $1{\cdot}0$; (b) $\Omega_R = 1{\cdot}00$, $\gamma = 0{\cdot}4$, $0{\cdot}6$, $1{\cdot}0$; (c) $\Omega_R = 1{\cdot}00$, $\gamma = 0{\cdot}8$, $1{\cdot}0$; (d) $\Omega_R = 1{\cdot}25$, $\gamma = 0{\cdot}4$, $0{\cdot}6$, $1{\cdot}0$; (e) $\Omega_R = 1{\cdot}25$, $\gamma = 0{\cdot}8$, $1{\cdot}0$; (f) $\Omega_R = 1{\cdot}50$, $\gamma = 0{\cdot}4$, $0{\cdot}6$, $1{\cdot}0$; (g) $\Omega_R = 1{\cdot}50$, $\gamma = 0{\cdot}8$, $1{\cdot}0$



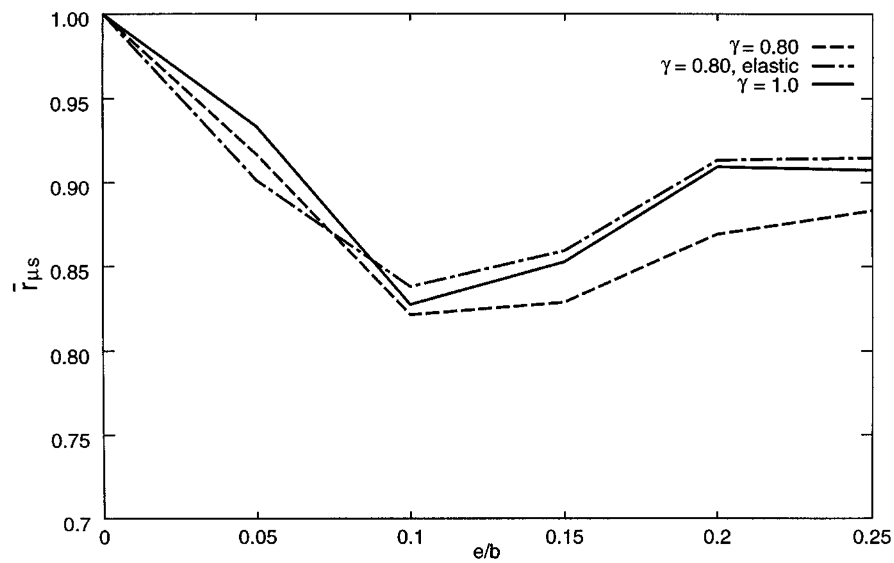
(c)



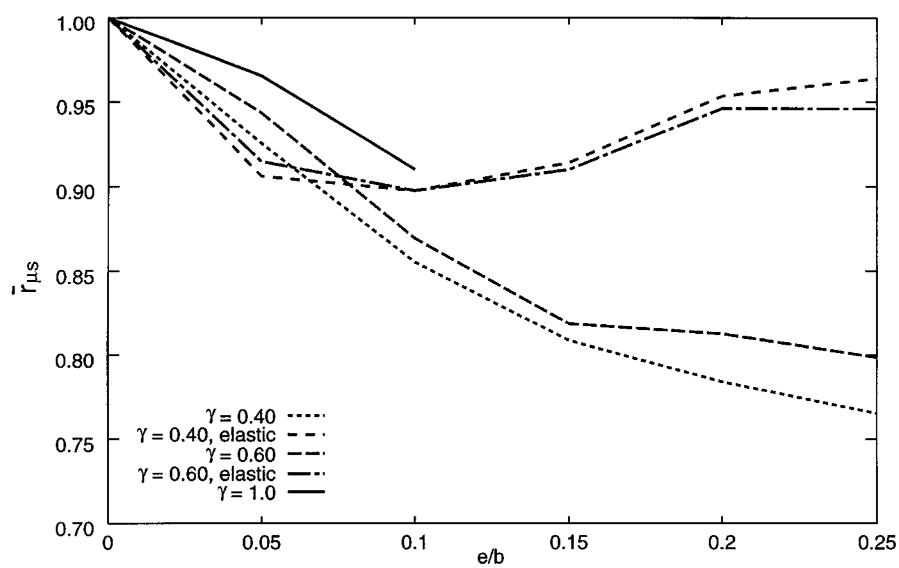
(d)

Figure 3. Continued

have been selected so as to conform to those used in studies by Correnza *et al.*[4,5] and Chandler *et al.*[6] In their studies, these authors have arrived at conclusions that are contradictory to those obtained from the present study. Comparison with their results is useful in explaining the reasons for this contradiction.



(e)



(f)

Figure 3. Continued

Dynamic analyses for response to El-Centro earthquake are carried out on the TB as well as TUB models A and B, following the procedures described earlier, except that the location of CM in the TUB models is retained at the geometric centre. The ductility demand in all planes of the TB model is given in column 2 of Table II. The demands in the flexible and stiff edge planes for

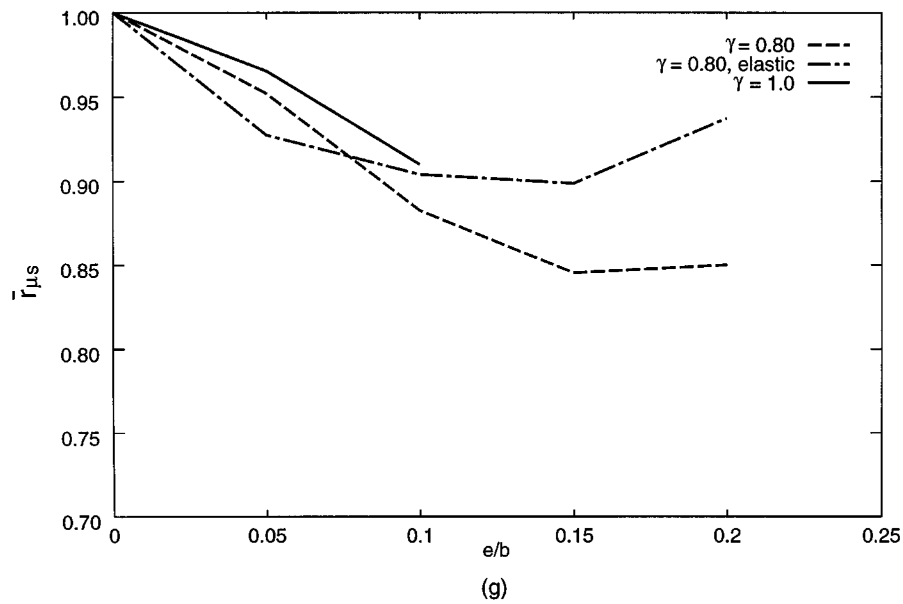


Figure 3. Continued

Table II. Effect of period on the torsional response of asymmetric building models

Periods	Ductility TB model μ_b	Ductility TBR model		Ductility TUB model no mass shift		Ductility TUB mass shifted $\pm 0.05b$		Ductility ratios			
		Flexible edge μ_{brf}	Stiff edge μ_{brs}	Flexible edge μ_{uf1}	Stiff edge μ_{us1}	Flexible edge μ_{uf2}	Stiff edge μ_{us2}	$\bar{r}_{\mu 1}$ $\frac{\mu_{uf2}}{\mu_b}$	$\bar{r}_{\mu 2}$ $\frac{\mu_{us2}}{\mu_b}$	$\bar{r}_{\mu 3}$ $\frac{\mu_{uf1}}{\mu_{brf}}$	$\bar{r}_{\mu 4}$ $\frac{\mu_{us1}}{\mu_{brs}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0.20	10·251	4·005	7·763	5·437	6·589	6·035	9·690	0·628	1·087	1·322	0·958
0·30	8·138	3·857	6·757	3·677	5·905	3·954	7·714	0·546	1·039	0·899	1·053
0·50	6·165	3·323	5·159	3·377	5·966	3·410	7·292	0·602	1·339	1·065	1·224
0·70	6·660	3·425	5·626	3·343	5·479	3·871	6·751	0·624	0·995	1·003	0·992
1·00	5·462	2·879	4·700	2·543	4·529	2·462	5·515	0·508	0·996	0·959	0·956
1·20	4·974	3·382	4·351	2·244	5·197	2·459	5·789	0·500	1·034	0·736	1·146
1·50	5·262	3·067	4·497	2·710	5·827	2·866	6·050	0·553	1·114	0·956	1·243
1·70	4·535	2·622	3·906	2·589	4·841	2·815	5·043	0·621	1·156	1·046	1·287
2·00	4·378	2·680	3·865	2·311	4·386	2·275	4·801	0·532	1·161	0·867	1·230
2·60	4·291	2·852	3·845	1·841	4·163	1·953	4·766	0·516	1·173	0·727	1·240
3·00	4·585	2·885	4·021	1·758	4·567	1·748	4·896	0·438	1·086	0·709	1·196

TUB model A are shown in columns 5 and 6, respectively. As described earlier, the design strengths of planes in the TUB models are determined by a procedure that accounts for the effect of accidental eccentricity. The design procedures can be considered adequate only if the response induced by the design earthquake meets the requirements of design even when accidental eccentricity effects are present in the calculated response. In other words, the dynamic analyses must include the effect of accidental eccentricity. As stated earlier, this is achieved by shifting the mass centre by a distance $\pm 0{\cdot}05b$ from the geometric centre and taking the larger of the ductility values determined from the analysis of the two TUB models so obtained. The revised ductility values determined as above are shown in columns 7 and 8 of Table II.

An important criterion in the seismic design is that the ductility demand in any resisting plane should not exceed the ductility capacity. Since the force modification factor R has been assumed to be approximately equal to the ductility capacity μ_t, it implies that the ductility demand should not, in this case, exceed 5. Now, it is known that the selection of $R = \mu_t$, which is based on the assumptions of equal total displacements in the elastic and inelastic models in a single-degree-of-freedom systems, does not always ensure that the ductility demand μ is equal to the target ductility μ_t. Consequently, even in the SDOF system, μ may be smaller or larger than μ_t. The TB model defined in this study is equivalent to a SDOF system, and as can be seen from the values presented in column 2 of Table II, μ is at times larger than μ_t and at other times smaller than μ_t. The purpose of torsional provisions is to ensure that the ductility demand imposed on the torsionally unbalanced system is not greater than that imposed on the corresponding balanced system. The design requirement can therefore be stated in the following alternative form: the ratio of TUB model response to TB model response, where the TUB model response has been calculated by including the effect of accidental eccentricity, should not exceed 1. Ratios $r_{\mu f}$ and $r_{\mu s}$ for the flexible and stiff edge planes, respectively, are shown in columns 9 and 10 of Table II, and plotted in Figures 4(a) and 4(b) as a function of period. It is seen that the ductility ratios do not vary significantly with period, and are always less than 1 for the flexible plane.

The procedure described in the preceding paragraph is a logical method of measuring the adequacy of torsional design provisions. Correnza and co-authors,[4, 5] however, specify a different method of evaluating the design procedures. In their method, they do not include the effect of accidental eccentricity in carrying out the dynamic analyses. Instead, they define a revised torsionally balanced reference model (TBR). In this model the strengths of resisting planes are obtained from a static procedure by applying the shear V_{y0} at $\pm 0{\cdot}01b$ from CR. The ductility demands in the flexible and stiff edge planes of TBR model are shown in columns 3 and 4 of Table II , respectively. The ductility ratios for flexible edge plane are obtained on dividing the value in column 5 (unmodified TUB) by those in column 3. Similarly, the ductility ratios for stiff edge plane are obtained on dividing column 6 by column 4. These ratios are shown in columns 11 and 12 of Table II, respectively, and are plotted in Figures 4(a) and 4(b). The plots show the flexible edge ductility ratio as being strongly dependent on the period and significantly higher than 1 in the short and medium period range. These conclusions are not correct and have been reached only because of the way in which a reference value has been defined. In fact, as seen from the results in column 7, the flexible edge ductility is less than 5, and hence satisfactory, in all cases except when the period is 0·2 s. Even for a period of 0·2 s the ductlity demand is significantly less than that in the torsionally balanced model.

The ductility ratios for TUB model B are also shown in Figures 4(a) and 4(b). They are only slightly better than those for model A. Apparently, torsional behaviour depends strongly on the

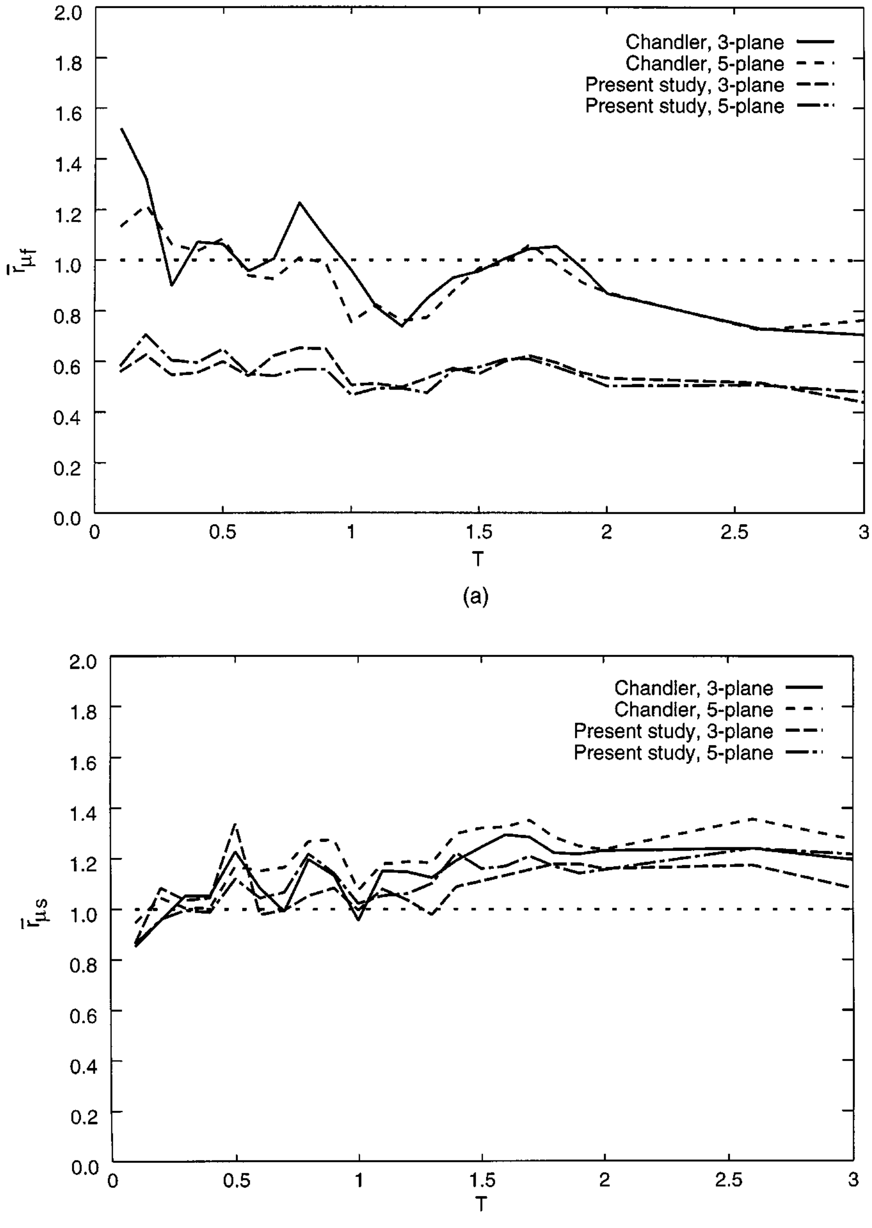


Figure 4. Average ductility ratios in 3 and 5 plane models, (a) flexible edge, (b) stiff edge

frequency ratio Ω_R, or the torsional stiffness, and for identical torsional stiffness, the responses of models with and without orthogonal planes are not significantly different.

PLASTIC MECHANISM ANALYSIS

In a series of recent studies Paulay[7,8] has examined the inelastic torsional response of single-storey building models. In these studies, buildings are classified into two categories: torsionally restrained and torsionally unrestrained. In a torsionally unrestrained building, there are no orthogonal planes and all but one of the parallel resisting planes yield during an earthquake. In a torsionally restrained building, all resisting planes parallel to the earthquake yield. However, at least some of the orthogonal planes remain elastic and contribute to torsional stiffness. One of the conclusions drawn in Paulay's studies is that the ductility demand in a critical plane of the unrestrained model, usually the one with the smallest yield displacement, is much higher than that for a torsionally restrained model. The above conclusion is contradictory to the results presented in the previous sections. The present authors have reviewed Paulay's earlier work[7] in Reference 2. In a more recent study,[8] Paulay makes several new observations and refinements. Results from Reference 8 that are relevant to the present study are reviewed here.

Consider the three-plane asymmetric building model as shown in Figure 5(a). According to Paulay's definition, this building model is torsionally unrestrained for an earthquake motion in the y-direction. The elastic spectral acceleration for the design earthquake corresponding to the uncoupled translation period of the building causes an elastic shear V_{ey} in the building. The associated torsionally balanced building would therefore be designed to have a total strength of $V_{y0} = V_{ey}/\mu_t$, where μ_t is the allowable ductility. This is based on the assumption usually made that the total displacement induced by the design earthquake in an inelastic building, comprising the yield displacement Δ_y and plastic displacement Δ_p, is equal to the displacement produced in the associated elastic building.

Assume that the strengths of individual planes in the torsionally unbalanced buildings are obtained from Equations (6)–(9) with $e_{d1} = e_{d2} = e$. As before, the yield forces are denoted by f_{y1}, f_{y2}, and f_{y3}. The corresponding yield displacements are Δ_{y1}, Δ_{y2}, and Δ_{y3}. Paulay reasons that when a plastic mechanism forms at ultimate load, the building should be in equilibrium under the action of seismic shear V_{y0} acting through the CM. For the model under study, CM coincides with the geometric centre. Hence, the forces imposed on planes 1 and 3 should be identical. Because the yield strengths of these two planes are not necessarily equal, only one of them could be yielding, while the other remains elastic. For example, if f_{y1} is greater than f_{y3}, plane 1 will still be elastic while planes 2 and 3 are yielding. Plane 1 cannot, in fact, yield unless plane 3 possesses post-yield stiffness and the displacement in that plane is sufficient to bring its strength up to f_{y1}. Beyond the first yield, all further deformations take place in planes 2 and 3. The floor deck, in effect, rotates about plane 1, imposing a high ductility demand on plane 3 which is farthest from the point of rotation.

To illustrate the above reasoning, consider a specific example of single-storey building model in which $e = 0{\cdot}1b$, $a/b = 0{\cdot}5$, $T = 1$ s and the design earthquake is El-Centro. The resisting planes in the building are assumed to be shear frames, and the diaphragm is considered infinitely rigid. The other characteristics are the same as presented earlier except that the force–displacement relationships for resisting planes are taken to be perfectly elasto-plastic. The El-Centro earthquake produces an elastic shear $V_{ey} = 2025$ kN, in the building. The ductility capacity is chosen as

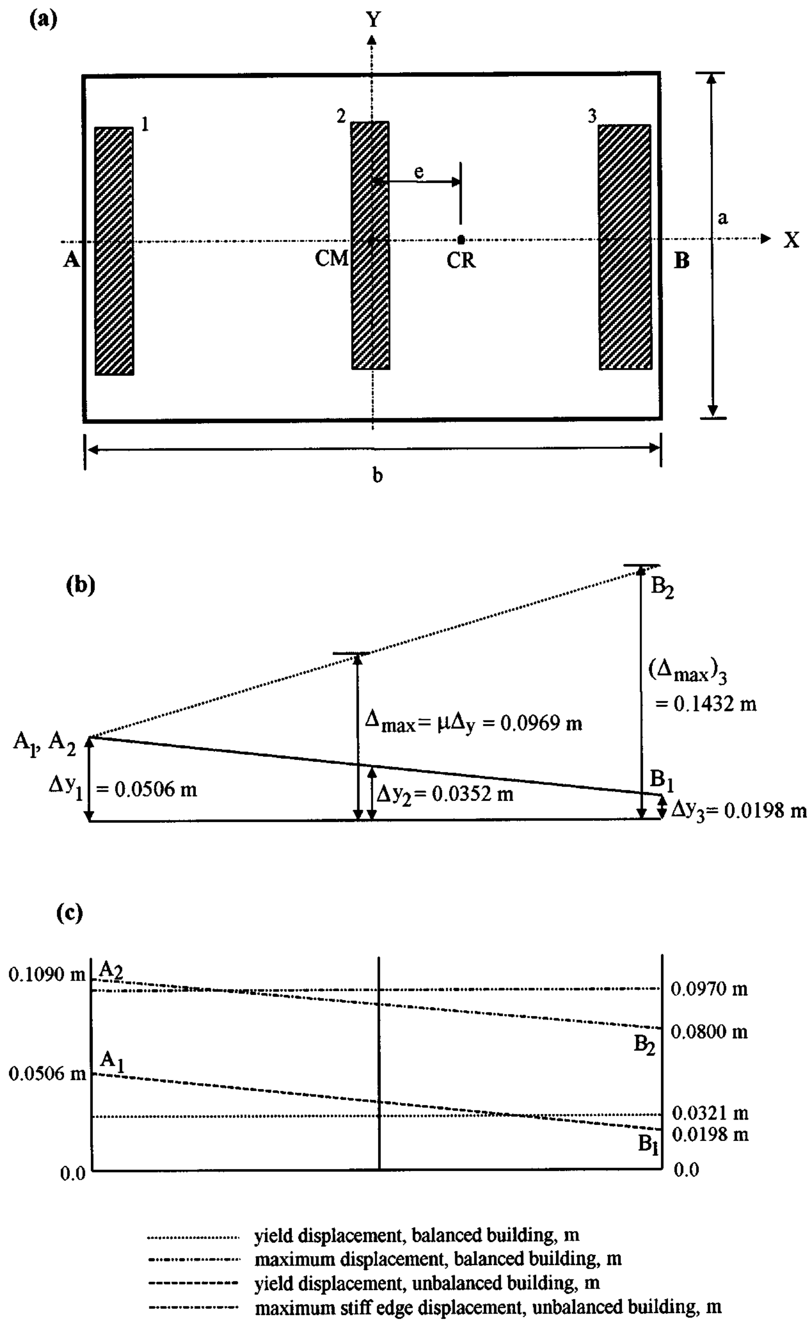


Figure 5. (a) Three-plane torsionally unrestrained building model, (b) Paulay's displacement pattern, (c) displacement pattern for El-Centro earthquake at the instant when stiff edge displacement is maximum

Table III. Characteristics and response of the three-plane torsionally balanced but unrestrained building model

Plane No.	Stiffness (kN/m)	Yield strength (kN)	Yield displacement (m)	Maximum total displacement (m)	Ductility
1	2026	65.1	0.0321	0.097	3.02
2	8580	275.4	0.0321	0.097	3.02
3	5185	166.5	0.0321	0.097	3.02

Table IV. Characteristics and response of the three-plane torsionally unbalanced and unrestrained building model

Plane No.	Stiffness (kN/m)	Yield strength (kN)	Yield displacement (m)	Maximum total displacement (m)	Ductility
1	2026	102·5	0·0506	0·121	2·390
2	8580	301·9	0·0352	0·097	2·757
3	5185	102·5	0·0198	0·080	4·044

$\mu_t = 4$, therefore the design shear is $V_{y0} = 506$ kN. For the torsionally balanced building the strengths of resisting planes are proportional to their stiffnesses.

The properties of the torsionally balanced building are shown in Table III. The maximum displacements produced in the building by El-Centro earthquake and the corresponding ductilities are also shown in that table. As would be expected, the displacements and ductilities are identical for all planes. The ductility value is in fact 3·02, which is lower than 4·0. The equal displacement assumption is obviously conservative for this earthquake and a period of $T = 1{\cdot}0$ s. However, in assessing the effect of torsion we will use 3·02 as the reference value.

The stiffnesses of the resisting planes in the torsionally unbalanced building are identical to those in the torsionally balanced building; the strengths derived from Equations (6)–(9) are shown in Table IV along with the yield displacements. Coincidentally, the yield strengths of planes 1 and 3 are equal in this case. However, for the purpose of our discussion we will assume that, as constructed, plane 1 has a strength slightly higher than the design value. The total displacements produced by the El-Centro earthquake are also shown in Table IV, along with the corresponding ductilities.

In accordance with the plastic mechanism analysis proposed by Paulay, the system yield displacement, Δ_y, for the model being studied is given by

$$\Delta_y = \frac{\sum f_{yi}}{\sum k_i} \tag{10}$$

$$= 0{\cdot}0321$$

At the formation of plastic mechanism, the displacement of plane 1 will be just below yield, that is, 0·0506 m. The displacement at the CM will be $3{\cdot}02 \times \Delta_y = 0{\cdot}0969$ and as shown in Figure 5(b),

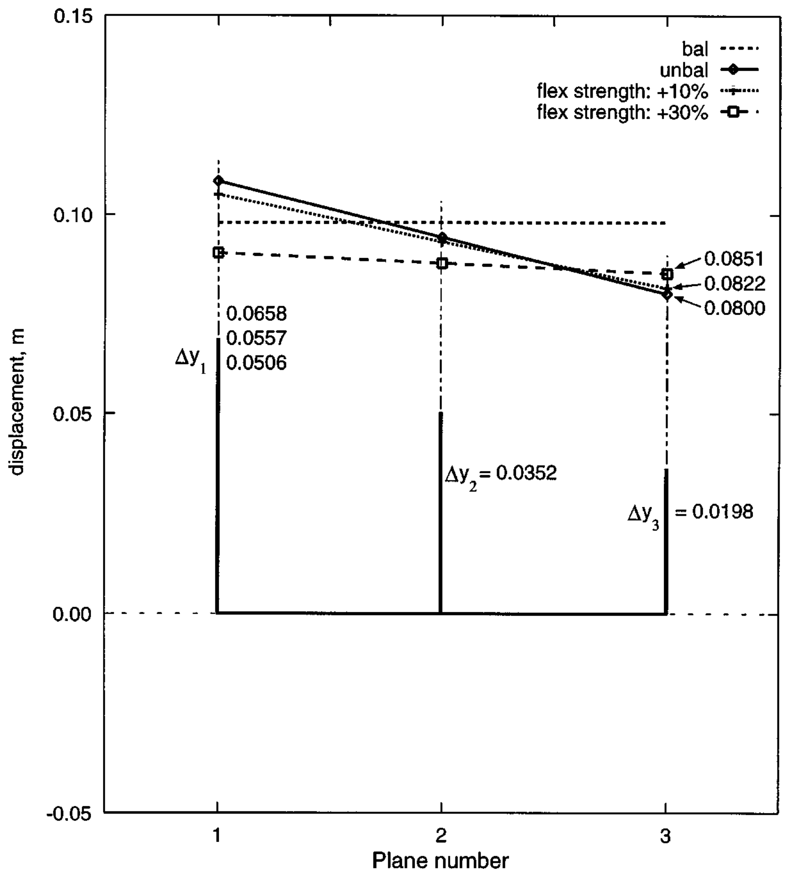


Figure 6. Displacement pattern for a three-plane torsionally unrestrained building model at the instant when stiff edge displacement is maximum, El-Centro earthquake, different strengths of the flexible edge plane

the displacement of plane 3 will be 0·1432. This gives a ductility demand of 7·23 for plane 3, which clearly is not acceptable.

The actual value of ductility demand obtained from the dynamic analysis is just 4.04. Obviously, Paulay's estimate is highly conservative. This is easily explained by observing Figure 5(c) which shows the displaced shape of the building axis at the time when the displacement of plane 3 is maximum. It is clear that rotation of the floor deck does not take place about the yield position of plane 1.

It may be argued that rotation about plane 1 could take place if the actual strength of that plane were higher than the design value. Therefore, the above analysis is repeated with plane 1 strength values 10 and 30 per cent higher than the design strength. The displaced shapes of the floor deck at the instant when the stiff edge plane achieves its maximum displacement are shown in Figure 6. The corresponding values of the plane 3 ductility are found to be 4·15 and 4·30, respectively, still much smaller than that predicted by Paulay.

The above conclusions were found to be true for other building configurations considered in the present study. The ductility demand in the critical plane reduces further, both when strain

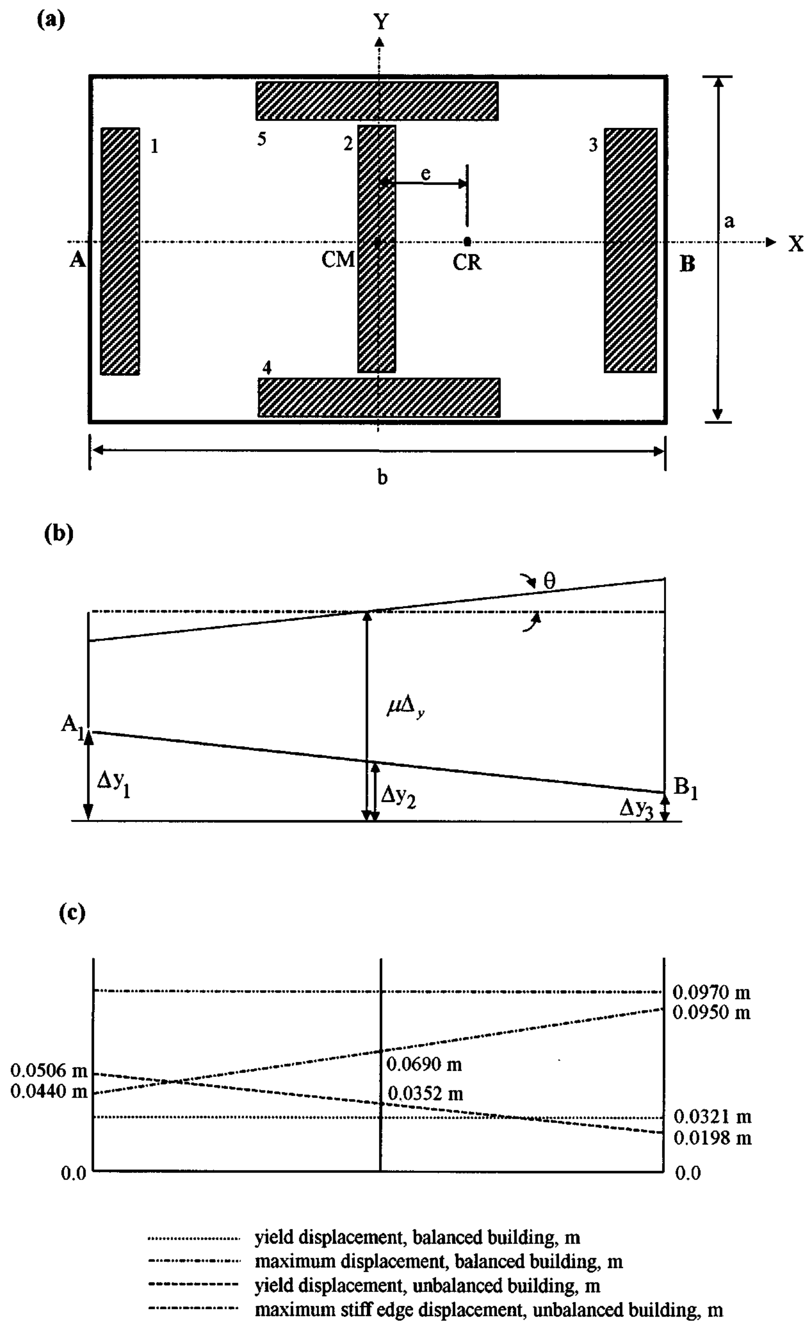


Figure 7. (a) Five-plane torsionally restrained building model, (b) Paulay's displacement pattern, (c) displacement pattern for El-Centro earthquake at the instant when stiff edge displacement is maximum

hardening is included and when the design eccentricity includes a provision for accidental torsion.

Figure 7(a) shows a torsionally restrained building in which it is assumed that orthogonal planes i.e. planes 4 and 5 remain elastic when the design earthquake strikes the building along the y-direction. The plan dimensions, the mass, the eccentricity and the frequency ratio are the same as those for the building in Figure 5(a). The characteristics of the torsionally balanced buildings as well as its response to El-Centro earthquake are shown in Table V. Similar information for the torsionally unbalanced building is presented in Table VI.

In accordance with Paulay's plastic mechanism method the ductility demand in Plane 3 is calculated as follows. First it is assumed that under the action of earthquake forces all planes yield and have a maximum displacement $\Delta_{\max}$ given by

$$\Delta_{\max} = \mu \Delta_y \tag{11}$$

where Δ_y is the system yield displacement given by equation (10), and μ is the system ductility, which we take as 3·02 in this case. At this stage there is an unbalanced moment in the system given by

$$M_t = (f_{y3} - f_{y1}) \frac{b}{2} \tag{12}$$

It is assumed that planes 4 and 5 that are orthogonal to the direction of earthquake are still elastic and resist the torsional moment. The torsional stiffness of these planes about the CM is given by

$$K_t = k_4 \left(\frac{a}{2}\right)^2 + k_5 \left(\frac{a}{2}\right)^2 \tag{13}$$

The resulting rotation of the deck about CM is

$$\theta = \frac{M_t}{K_t} \tag{14}$$

The total displacement of plane 3 is obtained from equations (11) and (13)

$$\Delta_{u3} = \mu \Delta_y + \frac{b}{2} \theta \tag{15}$$

Substitution of the characteristic values given in Table VI yields

$$\mu_3 = \frac{\Delta_{u3}}{\Delta_{y3}} \tag{16}$$

$$= 5{\cdot}67$$

The value obtained from a dynamic analysis is 4·8. It should be noted that this value is higher than that for a corresponding torsionally unrestrained building. The displacement of the building axis at an instant when stiff edge displacement is maximum, is shown in Figure 7(c). As explained earlier, the presence of orthogonal planes may at times increase the stiff edge ductility demand.

In summary, a large number of analytical studies carried out as part of the work on torsionally unrestrained building models, whose results are presented in the previous section, do not show

Table V. Characteristics and response of the five-plane torsionally balanced and restrained building model

Plane No.	Stiffness (kN/m)	Yield strength (N)	Yield displacement (m)	Maximum total displacement (m)	Ductility
1	381.6	12.25	0.0321	0.097	3.02
2	11869.0	381.1	0.0321	0.097	3.02
3	3539.9	113.6	0.0321	0.097	3.02
4	6579.8	Large			
5	6579.8	Large			

Table VI. Characteristics and response of the five-plane torsionally unbalanced and restrained building model

Plane No.	Stiffness (kN/m)	Yield strength (N)	Yield displacement (m)	Maximum total displacement (m)	Ductility
1	381·6	19·31	0·0506	0·123	2·43
2	11869·0	417·64	0·0352	0·096	2·73
3	3539·9	70·00	0·0198	0·095	4·80
4	6579·8	Large			
5	6579·8	Large			

trends predicted by Paulay's study. The lack of torsional restraint following yielding of resisting planes does not cause any significant problems. It is the overall torsional stiffness and not just the presence of orthogonal planes which is important in assessing the torsional behaviour of inelastic building systems. Paulay's plastic mechanism analysis is based on static equilibrium under the seismic forces induced by the inertia of the system. The inertial forces include a shear V as well as a torque M about the CM. The inertial torque is induced on account of coupling between the translational and rotational motion. Paulay includes only V in his analysis. The results obtained from such an analysis do not, therefore, resemble those obtained from a dynamic analysis which includes rotational inertia.

The trends observed in the present study also indicate that the results of previous studies, based on building models in which orthogonal planes remain elastic during an earthquake, provide conservative estimates of ductility demands at the stiff edge.

CONCLUSIONS

The torsional behaviour of asymmetric buildings subjected to earthquake motion is strongly influenced by the elastic torsional stiffness as measured by the ratio of uncoupled rotational

frequency to the uncoupled translational frequency. The torsional stiffness may arise from the planes parallel to the direction of earthquake, or as is most often the case, is a sum of contributions from planes both parallel and perpendicular to the direction of earthquake. When the torsional stiffness is contributed partly by the orthogonal planes, the ductility demand in the flexible planes is reduced although not by a large amount. On the other hand, the ductility demand in the stiff edge plane may be reduced or increased depending upon the value of the frequency ratio. In all cases the reduction or increase is fairly moderate. The trends noted above are accentuated when the orthogonal planes stay elastic during the earthquake motion.

The influence of orthogonal planes on ductility demands, as noted in the previous paragraphs is consistent for all periods.

ACKNOWLEDGEMENTS

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